Musterlösungen Test 1

1a) $f(x) = \ln(\ln(x))$

$$f'(x) = \underline{\underline{\frac{1}{\ln(x)} \cdot \frac{1}{x}}}$$ 2

b) $f(x) = e^{x \cdot \cos(x)}$

$$f'(x) = \underline{\underline{e^{x\cos(x)}\left\{\cos(x) + x\cdot(-\sin(x))\right\}}}$$ 2

2a) l'Hospital:

$$\lim_{x\to 1} \frac{\ln(x)}{\sin(\pi x)} \qquad \text{Typ } \frac{0}{0}$$

$$= \lim_{x\to 1} \frac{\frac{1}{x}}{\cos(\pi x)\cdot\pi} = \frac{1}{(-1)\pi} = \underline{\underline{-\frac{1}{\pi}}}$$ 2

2b) Grenzwert $a \in \mathbb{R}^2$ der Folge

$$a_n = \left(\underbrace{\frac{\ln(1+\frac{1}{n})}{\sin(\pi(1+\frac{1}{n}))}}_{=: a_{n,1}} , \underbrace{\sin\left(\frac{\pi n}{2n+2}\right)}_{=: a_{n,2}} \right)$$

- sei $x_n = 1 + \frac{1}{n}$, dann $\lim\limits_{n\to\infty} x_n = 1$, also $x_n \to 1$

nach 2a) ist $\lim\limits_{n\to\infty} \frac{\ln(x_n)}{\sin(\pi x_n)} = -\frac{1}{\pi}$

für alle Folgen $x_n \xrightarrow{n\to\infty} 1$

$$\Longrightarrow a_1 = \lim_{n\to\infty} a_{n,1} = \underline{\underline{-\frac{1}{\pi}}}$$ 1

• sei $x_n = \dfrac{\pi n}{2n+2} = \dfrac{\pi}{2+\underbrace{\frac{2}{n}}_{\to 0}} \longrightarrow \dfrac{\pi}{2}$

da sin(x) stetig ist, folgt

$$\lim_{n\to\infty} \underbrace{\sin(x_n)}_{=a_{n,2}} = \sin\left(\lim_{n\to\infty} x_n\right) = \sin\left(\frac{\pi}{2}\right) = \underline{\underline{1}}$$ 1

$\Rightarrow$ Grenzelement ist : $\underline{\underline{a = \left(-\frac{1}{\pi}, 1\right)}}$

3a) bestimme unbest. Integral

$$\int \underbrace{x}_{u} \underbrace{e^{-2x}}_{v'} dx = \underbrace{-\frac{1}{2} x e^{-2x}}_{uv} - \int 1 \cdot \left(-\frac{1}{2} e^{-2x}\right) dx$$

$v = -\frac{1}{2} e^{-2x}$

$$= -\frac{1}{2} x e^{-2x} + \frac{1}{2} \int e^{-2x} dx$$

$$\underline{\underline{= -\frac{1}{2} x e^{-2x} + \frac{1}{2}\left(-\frac{1}{2}\right) e^{-2x} + C}}$$ 3

3b) $$\int_0^\infty x e^{-2x} dx := \lim_{R\to\infty} \int_0^R x e^{-2x} dx$$

$$= \lim_{R\to\infty} -\frac{1}{2}\left\{ x e^{-2x} + \frac{1}{2} e^{-2x} \right\}\Big|_{x=0}^{x=R}$$

$$= \left(-\frac{1}{2}\right) \cdot \lim_{R\to\infty} \left(\left\{ R e^{-2R} + \underbrace{\frac{1}{2} e^{-2R}}_{\longrightarrow 0 \text{ für } R\to\infty} \right\} - \left\{ 0 + \frac{1}{2} \right\} \right)$$

$\underline{\underline{\text{NR}}}$ $$\boxed{\lim_{R\to\infty} R e^{-2R} = \lim_{R\to\infty} \frac{R}{e^{2R}} \overset{\text{l'Hosp.}}{=} \lim_{R\to\infty} \frac{1}{2 e^{2R}} = \underline{\underline{0}}}$$

$$\Rightarrow \int_0^\infty x e^{-2x} dx = \left(-\frac{1}{2}\right) \cdot \left(0 - \frac{1}{2}\right) = \underline{\underline{\frac{1}{4}}}$$ 3