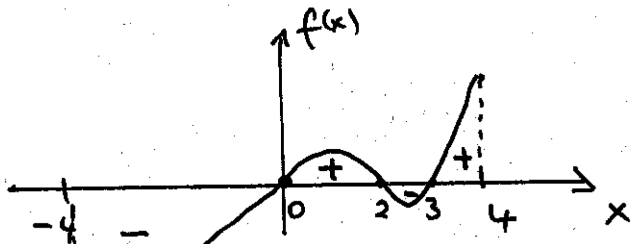
Musterlösungen Blatt 3

1a) $f(x) = x(x-2)(x-3)$



- Nullst.: $x_1 = 0,\ x_2 = 2,\ x_3 = 3$

$M = \{(x,y) \in \mathbb{R}^2 \mid x \in [4,4]$ und $\underbrace{0 \le y \le f(x)}\ \}$

$\Rightarrow$ notwendig $f(x) \ge 0$

- für den Flächeninhalt F_M von M folgt also

$$F_M = \int_0^2 f(x)\,dx + \int_3^4 f(x)\,dx$$

- $f(x) = x^3 - 5x^2 + 6x$

$\Rightarrow$ Stammfkt $\quad F(x) = \frac{1}{4}x^4 - \frac{5}{3}x^3 + 3x^2$

$$\Rightarrow F_M = \frac{1}{4}(16-0) - \frac{5}{3}(8-0) + 3(4-0)$$
$$+ \frac{1}{4}(256-81) - \frac{5}{3}(64-27) + 3(16-9)$$
$$= \underline{\underline{\frac{23}{4}}}$$

1b) <u>ges</u>: • Stammfkt. $F(x)$ zu $f(x) = |x-1|$
• $I = \int_0^3 |x-1|\,dx$

$$f(x) = \begin{cases} 1-x, & x \in (-\infty, 1) \\ x-1, & x \in [1, \infty) \end{cases}$$

$$\Rightarrow F(x) = \int_0^x f(t)\,dt$$

<u>1. Fall</u>: $x \in (-\infty, 1) \Rightarrow F(x) = \int_0^x (1-t)\,dt = t\Big|_0^x - \frac{1}{2}t^2\Big|_0^x$

$$\underline{\underline{= x - \frac{1}{2}x^2}}$$

2. Fall: $x \in [1,\infty) \Rightarrow F(x) = \int_0^1 (1-t)\,dt + \int_1^x (t-1)\,dt$

$$F(x) = \underbrace{1 - \tfrac{1}{2}1^2}_{\text{von Fall 1}} + \tfrac{1}{2}t^2\Big|_1^x - t\Big|_1^x$$

$$= \tfrac{1}{2} + \tfrac{1}{2}x^2 - \tfrac{1}{2} - (x-1)$$

$$= \underline{\underline{\tfrac{1}{2}x^2 - x + 1}}$$

$$\Rightarrow \text{Stammfkt.} \quad F(x) = \begin{cases} x - \tfrac{1}{2}x^2, & x \in (-\infty, 1) \\ \tfrac{1}{2}x^2 - x + 1, & x \in [1, \infty) \end{cases}$$

$$\Rightarrow I = \int_0^3 |x-1|\,dx = F(3) - F(0)$$

$$= \left(\tfrac{1}{2}\cdot 9 - 3 + 1\right) - 0 = \underline{\underline{\tfrac{5}{2}}}$$

2) unter Verwendung von $\boxed{\frac{d}{dx}(\ln(f(x))) = \frac{f'(x)}{f(x)}}$ (*)

berechne man:

a) $\int \tan(x)\,dx = \int \frac{\sin(x)}{\cos(x)}\,dx$

wähle: $f(x) = \cos(x) \Rightarrow f'(x) = -\sin(x)$

$$\overset{(*)}{\Rightarrow} \int \frac{f'(x)}{f(x)}\,dx = -\int \frac{\sin(x)}{\cos(x)}\,dx \overset{(*)}{=} \int \frac{d}{dx}\left\{\ln(\cos(x))\right\}dx$$

$$= \underline{\underline{\ln(\cos(x)) + C}} \Rightarrow \boxed{\int \tan(x)\,dx = -\ln(\cos(x)) + C}$$

b) $f(x) = 1 + x^2, \Rightarrow f'(x) = 2x$

$$\Rightarrow \int \frac{f'(x)}{f(x)}\,dx = \int \frac{2x}{1+x^2}\,dx = \int \frac{d}{dx}\left(\ln(1+x^2)\right)dx$$

$$= \ln(1+x^2) + C$$

$$\Rightarrow \int \frac{x}{1+x^2}\,dx = \underline{\underline{\frac{1}{2}\ln(1+x^2) + C}}$$

3a) <u>Vor</u>:

- $f:[a,b] \to \mathbb{R}$ stetig (1)
- $\int_a^b f(x)\,dx = 0$ (2)

<u>Beh</u>: $\exists\, c \in [a,b]: \quad f(c) = 0$ (3)

<u>Bew</u>:

- nach dem Mittelwertsatz der Integralrechnung $\exists\, \xi \in [a,b]$ mit $\int_a^b f(x) \cdot \underbrace{1}_{=g(x)>0} dx = f(\xi) \int_a^b \underbrace{1}_{g(x)} dx$

$$= f(\xi)(\cancel{b}-a)$$

- wegen (2) folgt $f(\xi) = 0 \Rightarrow$ (3) mit $c = \xi$ ∎

<u>3b)</u> Finde <u>Bsp</u> für $f, g : [0,1] \to \mathbb{R}$

$$\int_0^1 f(x)g(x)\,dx \neq \left\{\int_0^1 f(x)\,dx\right\} \cdot \left\{\int_0^1 g(x)\,dx\right\} \quad (4)$$

<u>Bsp</u>: $f(x) = \begin{cases} 1, & x \in [0, \frac{1}{2}] \\ 0, & x \in (\frac{1}{2}, 1] \end{cases}, \quad g(x) = \begin{cases} 0, & x \in [0, \frac{1}{2}] \\ 1, & x \in (\frac{1}{2}, 1] \end{cases}$

$$\Rightarrow f(x) \cdot g(x) = 0 \quad \forall\, x \in [0,1]$$

$$\Rightarrow \int_0^1 f(x) \cdot g(x)\,dx = 0$$

<u>aber</u>: $\int_0^1 f(x)\,dx = \int_0^{1/2} 1\,dx = \underline{\underline{\frac{1}{2}}}$

$$\int_0^1 g(x)\,dx = \int_{1/2}^1 1 \cdot dx = \underline{\underline{\frac{1}{2}}}$$

$$\Rightarrow \left\{\int_0^1 f(x)\,dx\right\}\left\{\int_0^1 g(x)\,dx\right\} = \frac{1}{4} \neq \int_0^1 f(x)g(x)\,dx = 0$$