Serie 3

4a) $?M : K(0,1) \longrightarrow \mathbb{C}_0 \setminus \overline{K}(1,1)$

Wählen:

$z_1 = -1 \quad , \quad M(z_1) = u_1 = 0$

$z_2 = 0 \quad , \quad M(z_2) = u_2 = i$

Und $u_3 = u_2^*$ Spiegelpkt. bzgl. $S(1,1)$

d.h. $(u_3 - 1)\overline{(\underbrace{u_2}_{=-i} - 1)} = 1$

also $-iu_3 - u_3 + i + 1 = 1$

$$u_3 = \frac{i+1}{2}$$

bei $M(\infty) = u_3$

$M(-1) = 0 = \dfrac{-1+b}{-c+d} = \dfrac{b-1}{d-c} = 0 \quad \Big| \; b = 1$

o.E.d.A. $a = 1$

$M(0) = \dfrac{0+1}{c \cdot 0 + d} = i = \dfrac{1}{d} \quad \Big| \; d = -i$

$M(\infty) = \dfrac{i+1}{2} = \dfrac{1}{c} \quad \Big| \; c = 1-i$

$$\underline{\underline{A}} = \begin{bmatrix} 1 & 1 \\ 1-i & -i \end{bmatrix}$$

Serie 3

5a)

$$[\log](i) = [\exp^{-1}(i)]$$
$$= \{ z \in \mathbb{C} : \exp(z) = i \}$$
$$= \{ (\tfrac{1}{2} + 2k)\pi i,\ k \in \mathbb{Z} \}$$

$$[i^i] = \{ \exp(iz) : z \in [\log](i) \}$$
$$\overset{\text{vgl. oben}}{=} \{ \exp(iz) : z \in \{ (\tfrac{1}{2} + 2k)\pi i,\ k \in \mathbb{Z} \} \}$$
$$= \{ \exp(-z) : z \in \{ (\tfrac{1}{2} + 2k)\pi,\ k \in \mathbb{Z} \} \}$$

$$[(\sqrt{2})^{1+2i}] = \{ \exp((1+2i)z) ; z \in [\log(\sqrt{2})] \}$$
$$= \{ \exp((1+2i)z) : z \in \{ \tfrac{1}{2}\ln 2 + 2k\pi i,\ k \in \mathbb{Z} \} \}$$
$$= \{ \exp\Big(\underbrace{(1+2i)\left(\frac{\ln 2}{2} + 2k\pi i\right)}_{(\frac{1}{2}\ln 2 + i\ln 2 + 2k\pi i - 4k\pi) \ldots} \Big) ;\ k \in \mathbb{Z} \}$$

5d)

$$\lim_{|b| \to \infty} |\sin(a+ib)| = \infty$$

$$\Longleftrightarrow\ \chi(\underbrace{\sin(a+ib)}_{= z_1}, \infty) \xrightarrow[\text{glm. in } a]{|b| \to \infty} 0$$

$$\chi^2(z_1, \infty)$$
$$[z_1 = \sin a \cosh b + i \cos a \sinh b]$$

$$= \left[\frac{2}{(1+|z_1|^2)^{\frac{1}{2}}} \right]^2 = \frac{4}{1 + \sinh^2(b) + \sin^2 a} \le \frac{4}{\cosh^2(b)}$$

$$|z_1|^2 = \sin^2 a \cosh^2(b) + \cos^2 a \sinh^2(b)$$
$$= \sin^2 a (1 + \sinh^2(b)) + \cos^2 a \sinh^2(b)$$
$$= \sin^2 a + \sinh^2(b)$$

Mit $0 \le \chi^2(z_1, \infty) \le \dfrac{4}{\cosh^2(b)}$ folgt alles mit $|b| \to \infty$