INVESTIGATION ON ONSET OF TURBULENCE FOR INCOMPRESSIBLE FLOWS IN ROTATING PIPES

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Abstract: There are considered general classes of boundary-pressure-driven flows of incompressible Newtonian fluids in three-dimensional pipes with known steady laminar realizations, where the focus is located on onset of the transition to turbulence. The basis of our investigations are the 3D-Navier-Stokes equations in unbounded pipes with nonslip conditions on the wall, at which the characteristic physical and geometrical quantities are subsumed in the kinetic Reynolds number and a parameter which involves the energetic ratio and the directions of the boundary-driven part and the pressure-driven part of the laminar flow. One gets the dimension-free velocity fields after convenient scaling . These fields are decomposed into the laminar flow and the remaining velocity (fulfilling homogeneous Dirichlet conditions on the boundary of the pipe and additionally required periodic conditions in the centerline-direction). The described properties enable the use of the Galerkin method with Stokes eigenfunctions in our studies. The Galerkin method is used to derive autonomous systems (S) of ordinary differential equations for the coefficients of the Galerkin approximation from the 3D-Navier-Stokes equations, where in an approximate sense S covers the essential physical properties of the velocity fields. Especially, the kinetic Reynolds number, the magnitude and the initial conditions, chosen as perturbations of the laminar velocity, are free parameters in S. The numerical studies of S are performed with the Dormand-Prince method, where the kinetic energies of the approximation are used for the evaluation of the calculations. The investigation of S yields approximations for critical (kinetic) Reynolds numbers of general pipe flows as well as unsteady approaches for velocities and pressures. Of special importance are results to time periodic strange numerical solutions of the 3D-Navier-Stokes equations, over a whole scale of boundary, driven and the pressure-driven flows. These solutions admit the treatment as bifurcations, and that S is especially qualified for further studies to transition and directly to the onset of turbulence. The final outcome of our investigations is the cognition that the direct use of the 3D-Navier-Stokes equations together with the Galerkin method on Stokes eigen functions is a very suitable offer for basic findings to the understanding of turbulence.

Key words: Navier-Stokes equations; stokes eigenfunctions; Galerkin methods; bifurcations; transition to turbulence

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INTRODUCTION

The pipe-flows of incompressible Newtonian fluids are well known, also owing to their simple geometry, as an old-established object of preference in theoretical and applied fluid-dynamic research, as well as ideal qualified examples for studies of the transition to turbulence and for investigations to extract further deterministic features from a random, fine-grained turbulent flow too[1-3].

It is the purpose of our investigations to check and clarify the possibilities of using low-dimensional Galerkin spaces defined by Stokes eigenfunctions for fact-finding the mechanism of the transition to turbulence in the non-stationary 3D-Navier-Stokes equations. Especially, our studies are targeted on the behavior of such ap-

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proximations in the vicinities of critical kinematic Reynolds numbers.

We explore a general class of scaled pipe flows of incompressible Newtonian fluids in unbounded pipes in $\mathbf{R}^3$, also referred to as rotating Hagen-Poiseuille flows, which can be described by the sum of a laminar boundary-driven Couette (angular momentum) flow $u_{L,C}$, a laminar pressure-driven Poiseuille flow $u_{L,P}$, and a time-dependent part u. The first and the second addenda are used to define the kinetic Reynolds number and a weighting parameter ψ for the energetic ratio and the direction of action of the boundary- and pressure-driven parts from u_L. Low-dimensional approximation spaces spanned by Stokes eigenfunctions with periodic conditions in the center-line-direction are applied on the direct numerical study of systems of Galerkin equations.

The essential notations and governing equations supplemented with initial and boundary conditions are given.

After convenient scaling, we decompose the velocity fields $\tilde{u}(t,x)$ into the laminar flow $u_L(x)$ and the remaining velocity $u(t, x)$ (fulfilling homogeneous Dirichlet conditions on the boundary of the pipe:) $\tilde{u}=u_L+u$. Additionally, periodic conditions in the center-line-direction are required for u. The pressure is decomposed similarly. The kinetic Reynolds numbers and a weighting parameter ψ for the energetic ratio and the direction of action of the boundary- and pressure-driven parts of u_L are also defined.

The usual function spaces for the mathematical treatment of the Navier-Stokes equations are introduced. In particular, we explain the Stokes operator and the Stokes eigenpairs (eigenvalues and eigenfunctions) $\{\lambda_j, w_j\}_{j=1}^{\infty}$ there. Afterwards we use the Stokes eigenpairs to explain the Galerkin approximations of $u_N := \sum_{j=1}^{N} g_j w_j$ the so-called weak solutions of the Navier-Stokes equations for the remaining velocities u. The Galerkin equations, as an autonomous system (S) of ordinary differential equations for the coefficients $g_j(t)$ of the Stokes eigenfunctions, are also given there.

The numerical method and the result are finally discussed. The dimension of the Galerkin-space $N=N(\lambda_{\max})=114$ is determined by a fixed (first) period $2\times l=2\times 2.69$ by a bound $\lambda_{\max}=50$ for the eigenvalues λ, where $\lambda_{\max}$ is taken in such a way, that the Galerkin-space includes two significant modes for the modification of the mean velocity both for the pure Couette-flow and the pure Poiseuille-flow.

We give some details to the determination of systems (S) of ordinary differential equations there. The systems (S) are solved numerically for several values of the parameters Re, ψ and a set of initial values $\{g_j(0)\}_{j=1}^{N}$ (small $u_{N,O} := \sum_{j=1}^{N} g_j(0) w_j$)), where the kinetic energy $E(t) = \sum_{j=1}^{N} g_j^2(t)$ of the Galerkin approximations u_N is used as a measure of turbulence and also as an indicator for periodic bifurcations[4].

1 BASIC NOTATIONS AND EQUATIONS

The non-stationary Navier-Stokes equations describe the time evolution of an incompressible Newtonian fluid. We are interested in the pipe flow, which means that the fluid is filling an open unbounded cylindrical domain $\Omega' := \{\mathbf{y}=(y_1, y_2, y_3)^{\mathrm{T}} \in \mathbf{R}^3 : \sqrt{y_2^2+y_3^2} < R\}$, where R with the unit m is the radius of the pipe.

The unknowns are the velocity field $\tilde{v}=\tilde{v}(\tau, y)$ (m/s), and the kinematic pressure $p'(\tau, y)$ ($\mathrm{m}^2/\mathrm{s}^2$), both depending on the actual time $\tau \in (0,\Theta)$ (s). One completes the problem by the usual input requirements: the kinematic viscosity (m^2/s); the initial conditions: $\tilde{v}(0,y) := \tilde{v}_Q(y)$; the boundary conditions: $\tilde{v}_R(\tau, y) := c_0(0, -y_3, y_2)^{\mathrm{T}}_{y_2^2+y_3^2=R_2}$ and vanishing external forces $f=0$. We regard only initial conditions of the kind $\tilde{v}_o = v_L + v_o$, where we denote by v_o disturbances of the laminar velocity fields v_L, with corresponding laminar pressures $p_L{}'$

$$v_L(y) := c_c(0, -y_3, y_2)^{\mathrm{T}} + c_p\chi R\left(1-\frac{y_2^2+y_3^2}{R^2}, 0, 0\right)^{\mathrm{T}} = v_{L,c} + v_{L,p}$$

$$p_L{}'(y):=\frac{c_c^2}{2}((y_2^2+y_3^2)-\frac{R^2}{2})-\nu c_p\chi\frac{4}{R}y_1 \quad (1)$$

We indicate here velocity parameters c_c, c_p (1/s) and the constant $\chi=\sqrt{3/2}$, which is chosen in such a way that for $c_c=c_p$ the velocity fields $v_{L,c}$ and $v_{L,p}$ result in the equal kinetic energy in a control volume. The characteristic (laminar) velocity v_{char} is introduced as the scale of the velocities by $v_{char}:=R\sqrt{c_c^2+c_p^2}$. Additionally the parameter $\psi\in[0,2\pi)$ is defined for all $v_{char}\neq 0$ as an indicator for the energetic ratio and the direction of action of the parts of the laminar velocity

$$\cos\psi:=\frac{c_c}{\sqrt{c_c^2+c_p^2}},\ \sin\psi:=\frac{c_p}{\sqrt{c_c^2+c_p^2}}$$
$$\psi\in[0,2\pi) \quad (2)$$

We denote the kinetic Reynolds number by Re: $Re=Re(c_c,c_p):=\frac{R^2}{\nu}\sqrt{c_c^2+c_p^2}$ and we use the following transformations of the variables

$$x=\frac{1}{R}y,\ t=\frac{\nu\tau}{R^2},\ \tilde{u}(\cdot,\cdot)=1\nu_{char}\tilde{v}(\cdot,\cdot)$$
$$\tilde{p}(\cdot,\cdot)=\frac{1}{\nu\sqrt{c_c^2+c_p^2}}p'(\cdot,\cdot) \quad (3)$$

to formulate the problem in dimension-free terms in the domain $\Omega:=\{x\in\mathbf{R}^3:\sqrt{x_2^2+x_3^2}<1\}$, where the scales are obvious. We get for the boundary conditions $\tilde{u}_{r=1}=\cos(\psi)(0,-x_3,x_2)^{\mathrm{T}}_{x_2^2+x_3^2=1}$ and for the scaled laminar fields, in Eq. (1) $u_L(x):=\cos(\psi)u_{L,c}+\sin(\psi)u_{L,p}$, with $u_{L,c}:=(0,-x_3,x_2)^{\mathrm{T}}$; $u_{L,p}:=(\chi(1-x_2^2-x_3^2),0,0)^{\mathrm{T}}$ and $p_L(x):=\frac{Re\cdot\cos^2(\psi)}{2}((x_2^2+x_3^2)-\frac{1}{2})-4\sin(\psi)\chi x_1$.

The velocities and pressures are handled by the use of splitting up formulas, similar to the initial conditions $\tilde{v}_0$. Therefore, we set $\tilde{u}(\cdot,\cdot)=u(\cdot,\cdot)+u_L(\cdot)$ and subsequently $p(\cdot,\cdot)=p_L(\cdot)+p(\cdot,\cdot)$.

Finally, periodic conditions for u and p are required instead of conditions in infinity, with l fix

$$u(t,x_1,x_2,x_3)=u(t,x_1+2l,x_2,x_3)$$
$$p(t,x_1,x_2,x_3)=p(t,x_1+2l,x_2,x_3)$$
$$l>1 \quad (4)$$

The Navier-Stokes initial-boundary value problem for the unknowns is given by Fig. 1, where some quantities are shown.

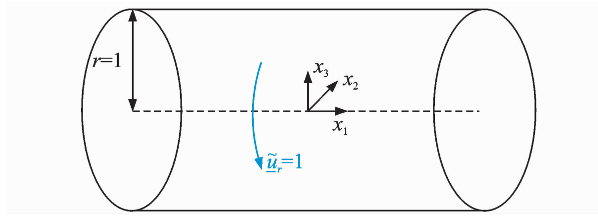

Fig. 1 Illustration of boundary movement and coordinates

We seek solutions $u(t,\boldsymbol{x})$ and $p(t,\boldsymbol{x})$ fulfilling in $(0,T)\times\Omega$: div $u=0$ and

$$\frac{\partial u}{\partial t}-\Delta_x u+Re(\sum_{j=1}^{3}u_j\frac{\partial u}{\partial x_j}+\cos(\psi)\sum_{j=1}^{3}u_{L,c,j}\frac{\partial u}{\partial x_j}+\sin(\psi)\sum_{j=1}^{3}u_{L,p,j}\frac{\partial u}{\partial x_j}+\cos(\psi)\sum_{j=1}^{3}u_j\frac{\partial u_{L,c}}{\partial x_j}+\sin(\psi)\sum_{j=1}^{3}u_j\frac{\partial u_{L,p}}{\partial x_j})+\nabla_x p=0 \quad (5)$$

with the initial and boundary conditions: $u(0,\boldsymbol{x})=u_0(\boldsymbol{x})=\tilde{u}_0(\cdot)-u_L(\cdot)$, Eq. (4) and $u(t,\boldsymbol{x})_{(x_2)^2+(x_3)^2=1}=0$.

2 EXPLANATION OF GALERKIN-APPROXIMATIONS

We restrict the domain Ω on the open bounded sub-domain T_l considering the presupposed periodic conditions in Eq. (4): $T_l:=\{\boldsymbol{x}\in\Omega:\boldsymbol{x}=(x_1,x_2,x_3)^{\mathrm{T}},\ |x_1|<l\}$, with $l>1$.

In what follows, we are going to make use of the successional abbreviations: We write $L_2(T_l)$ for the Hilbert space with the scalar product $(\boldsymbol{z},\boldsymbol{w}):=\int_{T_l}\boldsymbol{z}^{\mathrm{T}}(\boldsymbol{x})\boldsymbol{w}(\boldsymbol{x})\mathrm{d}\boldsymbol{x}$ and for the $L_2(T_l)$-subspace of fields with vanishing divergence S_l.

The S_l-subspace of vector fields with general derivatives with respect to the components of $\boldsymbol{x}$ (vanishing divergence and vanishing traces on $\partial T_l\cap\{\boldsymbol{x}:(x_2)^2+(x_3)^2=1\}$ will be denoted by S_l^1, where this space is endowed with the (Dirichlet-) scalar product $(\cdot,\cdot)_D$ defined by

$$(\boldsymbol{z},\boldsymbol{w})_D:=\sum_{k=1}^{3}\left(\frac{\partial\boldsymbol{z}}{\partial x_k},\frac{\partial\boldsymbol{w}}{\partial x_k}\right).$$

Finally, we denote $S_l^2:=S_l^1\cap W_2^2(T_l)$, where $W_2^2(T_l)$ represents the usual Sobolev space of vector valued functions.

The Stokes operator A plays a central role in the treatment of incompressible Navier-Stokes equations. It is termed by $A:=-\pi_l\Delta_x$, where π_l denotes the Leray-Helmholtz projection of $L_2(T_l)$ onto S_l as an linear operator $A: S_l^2 \subset S_l \rightarrow S_l$.

We remark that the Stokes operator is an operator with a pure point spectrum with the explicitly known eigenpairs $\{\lambda_j, w_j\}_{j=1}^{\infty}$, where all the eigenvalues λ_j of A are real and of finite multiplicity[5-6].

The associated eigenfunctions $\{w_j\}_{j=1}^{\infty}$ of the Stokes operator A (counted in multiplicity) are an orthogonal basis of S_l and S_l^1. We obtain also that $\{w_j\}_{j=1}^{\infty}$, with $1=(w_j, w_j)$, is a complete orthonormal system in S_l. In particular we have the eigenvalues $0<\lambda_1 \leqslant \lambda_2 \leqslant \cdots \leqslant \lambda_j \leqslant \cdots$ and $\lim_{j\rightarrow\infty}\bar{\lambda}_j=\infty$, at which the eigenpairs are satisfying the Stokes equations

$$
\begin{aligned}
&Aw_j=-\pi_l\Delta_x w_j:=\lambda_j w_j\\
&w_j \in D(A)=S_l^2 \quad \forall\, j=1,2,\cdots
\end{aligned}
\tag{6}
$$

Some notations for the explanation of the Galerkin equations will be declared in what follows. The Galerkin spaces as defined below are constituted by the use of the upper bound $\lambda_{\max}$ for the permitted eigenvalues. Let $N=N(\lambda_{\max})$ denote the number of eigenvalues of A with $\lambda_j \leqslant \lambda_{\max}$ (counted in multiplicity) and $N=N(\lambda_{\max}):=\sum_{\lambda_j\leqslant\lambda_{\max}} 1$. We apply the number N and $N=\dim(\operatorname{span}\{w_j\}_{j=1}^N)<\infty$ for the definition.

Definition: The (spatial) Galerkin space M_N as a subspace of S_N and the orthogonal projector P_N on M_N are stated by

$$
\begin{aligned}
&M_N:=\operatorname{span}\{w_j(x)\}_{j=1}^N \subset S_l\\
&P_N: S_l \rightarrow M_N (:\text{resp}: P_N: S_l^1 \rightarrow M_N)
\end{aligned}
\tag{7}
$$

For the actual formulation of the Galerkin approximations one uses the trilinearform $b(\cdot, \cdot, \cdot)$

$$
\begin{aligned}
&b(\boldsymbol{z}, q, \boldsymbol{w}):=(\sum_{j=1}^{3} z_j \frac{\partial q}{\partial x_j}, w)\\
&\forall\, (\boldsymbol{z}, q, \boldsymbol{w}) \in\\
&\quad L_2(0,T,S_l^1)\times L_2(0,T,S_l^1)\times L_2(0,T,S_l^1)
\end{aligned}
\tag{8}
$$

Using energetic apriori estimates and the theorem of Picard-Lindelöf one shows that there exists a unique Galerkin approximation $u_N=\sum_{j=1}^N g_j(t)w_j \in C^1(0,T,M_N)$ of a weak solution u of Eq. (5) for $u_0\in S_l$, where the coefficients $\{g_j\}_{j=1}^N$ of u_N are the solution[7] of the following initial value problem of the system of ordinary differential equations (S)

$$
\begin{aligned}
g_j=&\frac{\mathrm{d}}{\mathrm{d}t}(u_N, w_j)=-(u_N, w_j)_{\mathrm{D}}-\\
&Re\{b(u_N,u_N,w_j)+\cos(\psi)b(u_{\mathrm{L},c},u_N,w_j)+\\
&\sin(\psi)b(u_{\mathrm{L},p},u_N,w_j)+\cos(\psi)b(u_N,u_{\mathrm{L},c},w_j)+\\
&\sin(\psi)b(u_N,u_{\mathrm{L},p},w_j)\}=-\lambda_j g_j-\\
&Re\{\sum_{K,S=1}^N b_{j,K,S}g_K g_S+\cos(\psi)\sum_{K=1}^N q_{j,K}^{\mathrm{I}} g_K+\\
&\sin(\psi)\sum_{K=1}^N q_{j,K}^{\mathrm{II}} g_K+\cos(\psi)\sum_{K=1}^N r_{j,K}^{\mathrm{I}} g_K+\\
&\sin(\psi)\sum_{K=1}^N r_{j,K}^{\mathrm{II}} g_K\}\\
g_j(0)=&g_{j,O}:=(u_O,w_j) \quad j=1,2,\cdots,N\\
u_{N,O}=&\sum_{j=1}^N g_{j,O}w_j:=P_N u_O
\end{aligned}
\tag{9}
$$

Finally, the kinetic energy of our Galerkin approximation of the remaining velocity and its rate of change in time is termed by

$$
\begin{aligned}
&E(t):=\sum_{j=1}^N g_j^2(t)\\
&\dot{E}(t)=-2(\sum_{j=1}^N \lambda_j g_j^2(t)+\\
&\qquad Re(\cos(\psi)\sum_{j,K=1}^N q_{j,K}^{\mathrm{I}} g_j g_K+\\
&\qquad \sin(\psi)\sum_{j,K=1}^N q_{j,K}^{\mathrm{II}} g_j g_K))
\end{aligned}
\tag{10}
$$

3 NUMERICAL EXPERIMENTS AND RESULTS

The first steps in the numerical treatment are some preprocessings. One has to fix a period l and after that an upper bound $\lambda_{\max}$ for the eigenvalues of the considered Stokes eigenfunctions-respectively N as the dimension of the Galerkin space. The first preparation procedure generates the Stokes eigenfunctions $\{w_j\}_{j=1}^N$ and the required partial derivatives of these functions. In a second preparation procedure are calculated the coefficients of the ordinary differential equations (S). We use universalized tools of combined C- and MAPLE-routines together with implemented rules of general addition theorems in the form of allocation-lists.

The main step in the numerical treatment of the Galerkin approximations of the Navier-Stokes equations are studies of the numerical solutions of the systems according Eq. (9) in the dependence of the kinetic Reynolds number Re, the control parameter ψ, in Eq. (2), and the initial values $\{g_{j,0}\}_{j=1}^{N}$ as free parameters. There are implemented and used Dormand-Prince methods (DOPRI5)[8] with step size control. The initial value $g_{j,0}=0$, $\forall j=1,2,\cdots,N$, stands for the laminar velocities u_L as initial values. If the initial conditions $g_{j,0}$ have a distance smaller than $\rho<\sigma$ from the origin in $\mathbf{R}^N$, where $\rho=\rho(Re)$, our numerical solutions tend to the zero of $\mathbf{R}^N$. The reason is the asymptotic stability of the laminar flow in the sense of Ljapunov. The radius ρ of this ball of asymptotic stability depends nearly exponentially on Re. Generally, we use the kinetic energy $E(t)$, in Eq. (10), for all evaluations of the solutions.

It is worth to note that we also have procedures for the reconstruction of the pressures as well as tools for the calculation of mean values of the velocities u_N, of the Reynolds stresses and of the root-mean-square values of the fluctuating velocities. The comparison of the mean values of the velocities u_N with measurements shows at least satisfactory results. Whereas the agreement of calculations with the Reynolds stresses and the root-mean-square values of the fluctuating velocities is less satisfactory, however due to the small dimension of our approximation space most probably.

The critical kinetic Reynolds numbers for our calculations are

$$Re_{\text{crit}}^{\text{COUETTE}} \approx \infty,\ Re_{\text{crit}}^{\text{POIS}} \in (1\,400, 5\,000)$$

But the evaluation of our numerical investigations for general pipe flows yields a new significant feature. Much to our surprise we found time periodic non-laminar solutions of a constant kinetic energy $E(t,Re)$ at first fixed ψ-values out of a small ψ-interval ($\psi\approx\pi/3$) about a range of Re. There the rotation of the pipe acts stabilizing to the pressure-driven part of the velocity, like a bullet is stabilized by the spiral fluted barrel of a gun. This set of time periodic strange numerical solutions can be interpreted as the numerical result of stabilization by the physical property of angular momentum in the flows too.

From the mathematical point of view it seems that the time periodic non-laminar solutions are the first links in the bifurcation chain to turbulence[9].

Recently we are studying the time periodic bifurcation of u_L in the following direction: We use our periodic solutions for the general pipe flow as initial values $\{g_{j,0}\}_{j=1}^{N}$ in parameter investigations for $\psi\to\pi/2$ as the pure Poiseuille-flow limit.

The outcome of this will initiate a new direction of investigations: It consists of tracing our periodic solutions from the general pipe flow to the pure Poiseuille-flow.

4 CONCLUSIONS

In the present study, the transition of flows in pipes is investigated, where the bow is drawn from theoretical mathematics to numerical simulations. The flows are considered in an unique model with a unified description of general pipe flows. Some important findings as well as obtained results should be pointed out.

(1) The displayed method yields a suitable procedure to predict the critical Reynolds numbers for general pipe flows.

(2) The energy of the remaining velocity is an appropriate indicator for the evaluation of computational results as well as for the physical understanding of the numerical solutions.

(3) The calculated results for the mean velocities of the turbulent flow have proven satisfactory, but the accordance with measurements for quantities like the calculated Reynolds stresses is not a quarter as good, probably because of the

small dimension of our approximation space.

(4) The calculations reproduce the stabilization of rotating Hagen-Poiseuille flows by the influence of the angular momentum.

(5) Mathematical approximations with low-dimensional Galerkin spaces defined by Stokes eigenfunctions are qualified for investigations of the transition to turbulence in the non-stationary 3D-Navier-Stokes equations.

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