The conformal analogon of Calabi-Yau manifolds

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Abstract: Calabi-Yau manifolds are Riemannian manifolds with holonomy group $SU(m)$. They are Ricci-flat and Kähler and admit a 2-parameter family of parallel spinors.
In the talk we will discuss the conformal analogon of this situation. We consider the holonomy group of a conformal manifold $(M, [g])$, which is a subgroup of $SO(p+1, q+1)$ if the metric g has signature (p, q). We explain the relation between conformal holonomy groups and the existence of Einstein metrics in the conformal class as well as the relation between conformal holonomy groups and the existence of conformal Killing spinors. In particular, we describe Lorentzian manifolds (M, g) with conformal holonomy group in $SU(1, m)$, which can be viewed as the conformal analogon of Calabi-Yau manifolds. Such Lorentzian metrics g, known as Fefferman metrics, appear on S^1-bundles over strictly pseudoconvex CR spin manifolds and admit a 2-parameter family of conformal Killing spinors.