THE HEAT FLOW WITH A CRITICAL EXPONENTIAL NONLINEARITY

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On a domain $\Omega \subset \mathbb{R}^2$ jointly with Tobias Lamm and Frederic Robert we consider smooth solutions $u = u(t, x)$ to the equation

$$u_t e^{u^2} = \Delta u + \lambda u e^{u^2} \text{ in } [0, \infty[\times \Omega \tag{1}$$

with initial and boundary data

$$u(0) = u_0, \quad u = 0 \text{ on } [0, \infty[\times \partial\Omega, \tag{2}$$

and where the function $\lambda = \lambda(t)$ is determined so that the Dirichlet integral of u is preserved; that is,

$$\int_\Omega |\nabla u|^2 \, dx = \int_\Omega |\nabla u_0|^2 \, dx = \Lambda_0. \tag{3}$$

We demonstrate energy quantization effects for this flow, giving rise to existence results for the equation

$$-\Delta u = \lambda u e^{u^2} \text{ in } \Omega \tag{4}$$

in the "supercritical" range where $||\nabla u||_{L^2} = \Lambda_0 > 4\pi$.