THE HEAT FLOW WITH A CRITICAL EXPONENTIAL NONLINEARITY

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On a domain $\Omega \subset \mathbb{R}^2$ jointly with Tobias Lamm and Frederic Robert we consider smooth solutions $u = u(t, x)$ to the equation

(1) $u_t e^{u^2} = \Delta u + \lambda u e^{u^2}$ in $[0, \infty[ \times \Omega$

with initial and boundary data

(2) $u(0) = u_0$, $u = 0$ on $[0, \infty[ \times \partial \Omega,$

and where the function $\lambda = \lambda(t)$ is determined so that the Dirichlet integral of $u$ is preserved; that is,

(3) $\int_{\Omega} |\nabla u|^2 \ dx = \int_{\Omega} |\nabla u_0|^2 \ dx = \Lambda_0.$

We demonstrate energy quantization effects for this flow, giving rise to existence results for the equation

(4) $-\Delta u = \lambda u e^{u^2}$ in $\Omega$

in the “supercritical” range where $||\nabla u||_{L^2} = \Lambda_0 > 4\pi.$