Multiple solutions of the nonparametric Plateau problem in the Euclidean space $\mathbb{R}^p$ of arbitrary dimension

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Abstract

With the aid of Dirichlet's principle, we can minimize the area-functional for graphs in $\mathbb{R}^p$ defined on plane convex domains; here the dimension $p \geq 3$ is arbitrary. In the minimizing sequence we replace the graphs with embeddings of the unit disc via the uniformization method, and then we substitute them by the harmonic extensions of their boundary values. This is possible within the class of harmonic embeddings, due to a beautiful result by H. Kneser from 1926.

We calculate the first and second variation of the area-functional with higher codimensions and present the quasilinear nonparametric minimal surface system under Dirichlet boundary conditions, where uniqueness does not seem to prevail for $p \geq 4$. The constructed solutions above are stable in the sense that they possess a nonnegative second variation. However, we indicate by an example how to construct minimal graphs in $\mathbb{R}^5$ which are unstable.

Then we prove a mountain-pass lemma: An absolute minimizer together with a further strict relative minimizer of the area-functional possess a third minimal graph on the mountain-pass. In comparison with the Morse theory for Dirichlet's integral, we can consider critical points of the area-functional and apply the continuity theorem of M. Morse and C. Tompkins from 1941. Here we control the isothermal parameters along the admissible mountain-paths via the theory of nonlinear elliptic systems by E. Heinz.