Duality, regularity and uniqueness for BV-minimizers

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Abstract

For a smooth function $u\colon \Omega \to \mathbb{R}$ the n-dimensional area of its graph over a bounded domain $\Omega \subset \mathbb{R}^n$ is given by

$$\int_\Omega \sqrt{1+|Du|^2}\,dx\,.$$

A natural question is whether or not minimizers of this functional exist among all functions taking prescribed boundary values. It turns out that solutions of the least area problem exist only in a suitably generalized sense. This formulation is based on an extension of the original functional to the space of functions of bounded variation via relaxation, where attainment of the prescribed boundary values is not mandatory, but non-attainment is penalized. Consequently, such generalized minimizers do not need to be unique.

In my talk I will discuss similar convex variational integrals under a linear growth condition. After a short introduction to the dual problem in the sense of convex analysis I will explain the duality relations between generalized minimizers and the dual solution. The duality relations can be interpreted as mutual representation formulas, and in particular they allow to deduce statements on uniqueness and regularity for generalized minimizers. The results presented in this talk are based on a joined project with Thomas Schmidt (Erlangen).